Practice 6

Topic: ***Research of special points on the phase plane***

The example Let a dynamic system is described by a system of equations in the state-space:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \qquad (*)$$

where $A = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}, \; B = \begin{vmatrix} -5 \\ 1 \end{vmatrix}, \; C = \begin{vmatrix} 1 & 1 \end{vmatrix}.$

You should define a type of transition process and define what special point (stationary point) corresponds the phase portrait of the researched system; show geometrical interpretation.

Algorithm and solution

1. We obtain own numbers of a matrix *A:*

$$\det(A - \lambda I) = 0.$$

$$\det \begin{vmatrix} (-2-\lambda) & 1 \\ 1 & (-2-\lambda) \end{vmatrix} = 0;$$

$$(-2-\lambda)^2 - 1 = 0$$

$$\lambda_1 = -1; \quad \lambda_2 = -3.$$

Hence, the movement of the given dynamic system asymptotically is steady across Lyapunov as real parts of roots are negative, i.e. $Re\, \lambda_i(A) < 0$.

Geometrical interpretation:

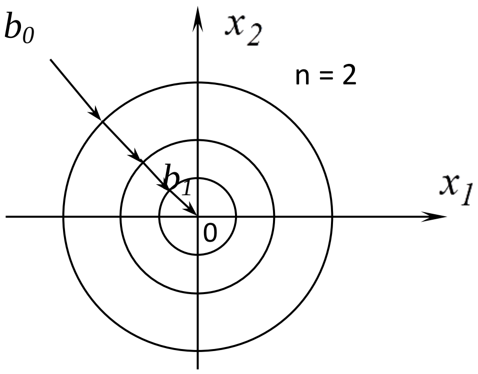

Fig. 1- The movement of the given dynamic system asymptotically is steady across Lyapunov

The roots of characteristic equation real and negative, therefore, transient process is monotonous and steady.

In fig. 2 the arrangement of roots of characteristic equation of the researched system and the transient process corresponding to them are presented.

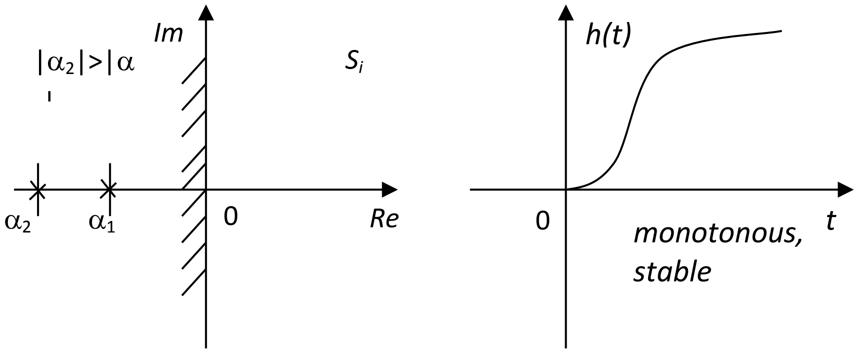

Fig. 2 – Arrangement of roots and the process corresponding to them transient process

In this case the phase portrait corresponds to a special point, *stable knot* (fig.6.12). Here the straight line is a degenerated trajectory $e^{-\alpha t}$

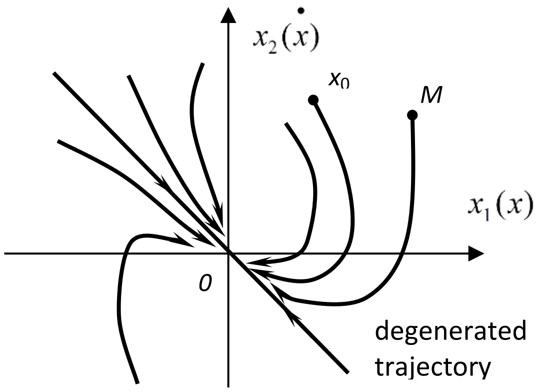

Fig. 3 – The Special point is a steady node

Task Let a dynamic system is described by a system of equations in the state-space:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \qquad (*)$$

where the matrixes *A, B* and *C* are set below by variants.

You should define a type of transition process and define what special point (stationary point) corresponds the phase portrait of the researched system; show geometrical interpretation.

Variants:

1)

$$A=\begin{vmatrix}-3 & 4\\ 6 & -5\end{vmatrix},\ B=\begin{vmatrix}2\\ -2\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

2)

$$A=\begin{vmatrix}-2 & 4\\ -1 & -5\end{vmatrix},\ B=\begin{vmatrix}-1\\ 1\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

3)

$$A=\begin{vmatrix}1 & -1\\ 2,5 & 4\end{vmatrix},\ B=\begin{vmatrix}-2\\ -2\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

4)

$$A=\begin{vmatrix}2 & 4\\ -1 & 5\end{vmatrix},\ B=\begin{vmatrix}7\\ 3\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

5)

$$A=\begin{vmatrix}2 & 6\\ 8 & 4\end{vmatrix},\ B=\begin{vmatrix}-3\\ 3\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

6)

$$A=\begin{vmatrix}9 & 9\\ 2 & 6\end{vmatrix},\ B=\begin{vmatrix}1\\ 6\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

7)

$$A=\begin{vmatrix}0 & 9\\ -1 & 0\end{vmatrix},\ B=\begin{vmatrix}5\\ 1\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

8)

$$A=\begin{vmatrix}-1 & -1\\ 2,5 & -4\end{vmatrix},\ B=\begin{vmatrix}-3\\ -2\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

9)

$$A=\begin{vmatrix}1 & -1\\ 7 & 9\end{vmatrix},\ B=\begin{vmatrix}2\\ 3\end{vmatrix},\ C=\begin{vmatrix}1\\ 1\end{vmatrix}.$$

10)

$$A = \begin{vmatrix} -4 & 3 \\ 3 & -4 \end{vmatrix}, \; B = \begin{vmatrix} -1 \\ 2 \end{vmatrix}, \; C = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$

11)

$$A = \begin{vmatrix} 2 & -5 \\ 1 & 6 \end{vmatrix}, \; B = \begin{vmatrix} 8 \\ -3 \end{vmatrix}, \; C = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$

12)

$$A = \begin{vmatrix} 4 & -7 \\ 2 & 8 \end{vmatrix}, \; B = \begin{vmatrix} -5 \\ 2 \end{vmatrix}, \; C = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$